CHAPTER: 3 and 4

MATRICES AND DETERMINANTS

IMPORTANT POINTS TO REMEMBER

Matrix: **It** is an ordered rectangular arrangement of numbers (or functions). The numbers (or functions) are called the **elements** of the matrix. Horizontal line of elements is **row** of matrix. Vertical line of elements is **column** of matrix.

Numbers written in the horizontal line form a row of the matrix. Number written in the vertical line form a column of the matrix.

Order of Matrix with '*m*' rows and '*n*' columns is $m \times n$ *(read as m by n).*

Types of Matrices

- A **row matrix** has only one row (order:$1\times n$)
- A **column matrix** has only one column (order: $m\times 1$)
- A **square matrix** has number of rows equal to number of columns (order: $m \times m$ or $n \times n$.)
- A **diagonal matrix** is a square matrix with all non-diagonal elements equal to zero and diagonal elements not all zeroes.
- A **scalar matrix** is a diagonal matrix in which all diagonal elements are equal.
- An **identity matrix** is a scalar matrix in which each diagonal element is 1 (unity).
- **A zero matrix** or **null matrix** is the matrix having all elements zero.

- **Equal matrices:** two matrices A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]$ $m \times n$ are equal if

  (a) Both have same order

  (b) $a_{ij} = b_{ij} \forall$ i and j

Operations on matrices

- Two matrices can be added or subtracted, if both have same order.

- If A= $[a_{ij}]_{m \times n}$ and B =$[b_{ij}]_{m \times n}$, then

  A $\pm$ B = $[a_{ij} \pm b_{ij}]_{m \times n}$

- $\lambda A = [\lambda a_{ij}]_{m \times n}$ where λ is a scalar

- Two matrices A and B can be multiplied if number of columns in A is equal to number of rows in B.

  If $A = [a_{ij}]_{m \times n}$ and $[b_{jk}]_{n \times p}$

  Then $AB = [c_{ik}]_{m \times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

  Properties

- If A, B and C are matrices of same order, then

  (i) A+B = B+A

  (ii) (A+B)+C= A+(B+C)

  (iii) A+O = O+A=A

  (iv) A+(-A) = O

- If A, B and C are matrices and λ is any scalar, then

  a. $AB \neq BA$

  b. (AB) C = A(BC)

  c. A(B+C) = AB+AC

  d. AB=O need not necessarily imply A=O or B=O

  e. λ (AB)= (λA) B=A (λB)

Transpose of a Matrix: Let A be any matrix. Interchange rows and columns of A. The new matrix so obtained is transpose of A donated by A'/A^T.

[order of A = m × n ⇨ order of A' = n × m]

Properties of transpose matrices A and B are:

(i) $(A')' = A$

(ii) $(kA)' = kA'$ (k= constant)

(iii) $(A + B)' = A' + B'$

(iv) $(AB)' = B'A'$

Symmetric Matrix and Skew-Symmetric matrix

- A square matrix $A = [a_{ij}]$ is symmetric if $A' = A$i.e. .$a_{ij} = a_{ji}$∀i and j
- A square matrix A= $[a_{ij}]$ is skew-symmetric if $A' = -A$ i.e. $a_{ij} = - a_{ji}$∀ i and j (All diagonal elements are zero in skew-symmetric matrix)

Determinant: to every square matrix A= $[a_{ij}]$ of order n×n, we can associate a number (real or complex). This is called determinant of A (det A or $|A|$).

Properties of Determinants

I) $|AB| = |A|\,|B|$

II) $|A'| = |A|$

III) If we interchange any two rows (or columns), sign of $|A|$ changes.

IV) Value of $|A|$ is zero, if any two rows or columns in A are identical (or proportional).

V) $\begin{vmatrix} a+b & x \\ c+d & y \end{vmatrix} = \begin{vmatrix} a & x \\ c & y \end{vmatrix} + \begin{vmatrix} b & x \\ d & y \end{vmatrix}$

VI) $R_i \to R_i \pm aR_j$ or $C_i \to C_i \pm bC_j$ does not alter the value of $|A|$.

VII) $|k\,A|_{n\times n} = k^n |A|_{n\times n}$ (k= scalar)

VIII) $K\,|A|$ means multiplying only one row (or column) by k.

IX) Area of triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear if area of triangle is zero

Minors and Cofactors

- Minor (M_{ij}) of a_{ij} in Matrix A is the determinant of order (n-1) obtained by leaving i^{th} row and j^{th} column of A.
- Cofactor of a_{ij} , $A_{ij} = (-1)^{i+j} M_{ij}$

Adjoint of a Square Matrix

adj A= transpose of the square matrix A whose elements have been replaced by their cofactors.

Properties of adj A: For any square matrix A of order n:

(i) $A(adj\ A) = (adj\ A)\ A = |A|\ I$

(ii) $|adj\ A| = |A|^{n-1}$

(iii) $adj(AB) = (adj\ B)\ (adj\ A)$.

(iv) $|k\ adj\ A| = k^n\ |A|^{n-1}$.

Singular Matrix: A square matrix A is singular if $|A| = 0$.

Inverse of a Matrix: An inverse of a square matrix exists if and only if it is non-singular.

$A^{-1} = \frac{1}{|A|}$ adj A

Properties of Inverse matrix

(i) $AA^{-1} = A^{-1}A = I$

(ii) $(A^{-1})^{-1} = A$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A')^{-1} = (A^{-1})'$

(v) $|A'| = \frac{1}{|A|}$, $|A| \neq 0$

Solution of system of equations using matrices:

If $AX = B$ is a matrix equation, then

$AX=B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow IX = A^{-1}B \Rightarrow X = A^{-1}B$ gives the solution.

Criterion of consistency of system of linear equations

(i) If $|A| \neq 0$, system is consistent and has a unique solution.

(ii) If $|A| = 0$ and $(adj\ A)\ B \neq 0$, then the system AX=B is inconsistent and has no solution.

(iii) If $|A| = 0$ and $(adj\ A)\ B = 0$ then system is consistent and has infinitely many solutions.

ONE MARK QUESTIONS

1. If $[1\ x\ 1]\begin{bmatrix}1 & 3 & 2\\0 & 5 & 1\\0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix} = 0$,then What is the value of x?

2. For what value of λ, the matrix A is a singular matrix where

$$A = \begin{bmatrix}1 & 3 & \lambda+2\\2 & 4 & 8\\3 & 5 & 10\end{bmatrix}$$

3. Find the value of A^2, if

$$A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\a & b & -1\end{bmatrix}$$

4. If $A = \begin{bmatrix}a & b\\b & a\end{bmatrix}$ and $A^2 = \begin{bmatrix}\alpha & \beta\\\beta & \alpha\end{bmatrix}$, then find the value of $\propto$ $and\ \beta$.

5. If A is a square matrix such that $A^2 = I$, then write the value of $(A - I)^3 + (A + I)^3 - 7$ A in simplest form.

6. Write the value of Δ, if

$$\Delta = \begin{vmatrix}x+y & y+z & z+x\\z & x & y\\-3 & -3 & -3\end{vmatrix}$$

7. If $\begin{bmatrix}x-y & z\\2x-y & w\end{bmatrix} = \begin{bmatrix}-1 & 4\\0 & 5\end{bmatrix}$,find the value of x+y.

8. If A is a 3 × 3 matrix, $|A| \neq 0$ and $|3A| = K|A|$, then write the value of K.

9. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then write the value of x.

10. Matrix $A = \begin{bmatrix} 0 & 2a & -2 \\ 3 & 1 & 3 \\ 3b & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the value of a and b.

11. For any 2 × 2 matrix A, if A (adjoint A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find $|A|$.

12. Find X, if A + X = I, where

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 3 & 4 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

13. If $U = [2 \quad -3 \quad 4], V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$,X= $[0 \quad 2 \quad 3]$ and $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, then find UV+XY.

14. If $\begin{bmatrix} 2 & -3 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -9 \\ 16 & 15 \end{bmatrix}$

write the equation after applying elementary column transformation $C_2 \to C_2 + 2C_1$

15. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then find the value of A^3.

16. Find the value of $a_{23} + a_{32}$ in the matrix

$A = [a_{ij}]_{3\times3}$where $a_{ij} = \begin{cases} |2\mathrm{i} - \mathrm{j}| & \text{if } \mathrm{i} > j \\ -\mathrm{i} + 2\mathrm{j} + 3 & \text{if } \mathrm{i} < j \end{cases}$

17. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then find $|A^2|$.

18. For what value of x, is the matrix

$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & x & -3 \\ 2 & 3 & 0 \end{bmatrix}$ a skew-symmetric matrix

19. If $A = \begin{bmatrix} \sin 15^\circ & \cos 15^\circ \\ -\sin 75^\circ & \cos 75^\circ \end{bmatrix}$, then evaluate $|A|$.

20. If A is a square matrix, expressed as A= X + Y where X is symmetric and Y is skew-symmetric, then write the values of X and Y.

21. Write a matrix of order 3 × 3 which is both symmetric and skew-symmetric matrix.

22. What positive value of x makes the following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \quad \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

23. $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, find the value of $5A_{31} + 3A_{32} + 8A_{33}$.

24. If $A = \begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, find $|A\,(adjA)|$

25. Find the minimum value of. $2\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + sin\theta & 1 \\ 1 & 1 & 1 + cos\theta \end{vmatrix}$

26. If A and B are square matrices of order 3 and $|A| = 5$ and $|B| = 3$, then find the value of $|3AB|$.

27. Evaluate $\begin{vmatrix} 3 + 2i & -6i \\ 2i & 3 - 2i \end{vmatrix}$, $i = \sqrt{-1}$

28. Without expanding, find the value of $\begin{vmatrix} cosec^2\theta & cot^2\theta & 1 \\ cot^2\theta & cosec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix}$

29. Using determinants, find the equation of line passing through (0, 3) and (1,1).

30. If A be any square matrix of order 3 × 3 and $|A| = 5,$ then find the value of $|adj\ (adjA)|$

31. What is the number of all possible matrices of order 2 × 3 with each entry 0,1 or 2.

32. Given a square matrix A of order 3 × 3 such that $|A|$=12, find the value of $|A\ adj\ A|$

33. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ find $|(A^{-1})^{-1}|$

34. If $A = [-1 \quad 2 \quad 3]$ and $B = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ find $|AB|$

35. Find $|A\ (adjoint\ A)|$ and $|adjoint\ A|$, if $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

TWO MARK QUESTIONS

1. Construct a matrix of order 2 × 3, whose elements are given by

(a) aij = $\frac{(i-2j)^2}{2}$ (b) aij = $\frac{|-2i+j|}{3}$

2. If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of on equilateral triangle with each side equal to a units, than prove that

$$\begin{bmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{bmatrix}^2 = 3a^4$$

3. Prove that the diagonal elements of a skew-symmetric matrix are all zero.

4. If $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Find the valve of x – y

5. If A and B are skew symmetric matrices of the same order prove that AB + BA is symmetric matrix.

6. Without expending prove that $\begin{bmatrix} o & p-q & p-r \\ q-p & o & q-r \\ r-p & r-q & o \end{bmatrix} = 0$

7. Let A= $\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$ Prove that A+A′ is symmetric matrix.

8. If A= $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ and B=[1 2 3], Verify (AB)′ = B′A′

9. If A= $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and C= $\begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

Find AB-AC.

10. If A= $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ Find the determinant of A^2-2A

11. Without expanding, evaluate $\begin{bmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{bmatrix}$

12. If $D_1 = \begin{vmatrix} a & b & c \\ x & y & z \\ l & m & n \end{vmatrix}$ and $D_2 = \begin{vmatrix} m & -b & y \\ -l & a & -x \\ n & -c & z \end{vmatrix}$ evaluate D_1+D_2.

13. If A is a skew symmetric matrix of odd order, then prove that |A|= 0

14. Write the minors and co-factors of each element of the first column of the matrix A

$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$

15. Find x and y, if $\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

16. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, find matrix 'C', such that 2A+3C=5B

17. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ find x such that $A^2 = B$.

18. Construct a matrix of order 3x2, whose elements a_{ij} given by

$$a_{ij} = \begin{cases} 2i-3j, & i \geq j \\ 3i+j, & i < j \end{cases}$$

4 MARK QUESTIONS

1. If $\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$, then prove that $\frac{a}{a-x} + \frac{b}{b-y} + \frac{c}{c-z} = 2$

2. If a ≠ b ≠ c, find the value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

3. Using properties of determinants, show that

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 0$$

4. Find the value of $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

5. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$, show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

6. Find the matrix X so that $X\begin{bmatrix}1 & 2\\ 5 & 3\end{bmatrix} = \begin{bmatrix}5 & 10\\ 2 & 0\end{bmatrix}$

7. If $A = \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, verify that $A^2 - 4A - 5I = 0$.

8. Using elementary transformations find the inverse of the matrix

$A = \begin{bmatrix}2 & 1\\ 4 & 7\end{bmatrix}$

9. If $A = \begin{bmatrix}x & -2\\ 3 & 7\end{bmatrix}$ and $A^{-1} = \begin{bmatrix}\frac{7}{34} & \frac{1}{17}\\ -\frac{3}{34} & \frac{2}{17}\end{bmatrix}$, then find the value of x.

10. If $A = \begin{bmatrix}2 & -3\\ 0 & 1\end{bmatrix}$, find B, such that $4A^{-1} + B = A^2$

11. If $A = \begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}, 10B = \begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \propto\\ 1 & -2 & 3\end{bmatrix}$ and B=A^{-1}, then find the value of ∝.

12. Find the value of X, such that $A^2 - 5A + 4I + X = 0$, if $A = \begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

13. If $A = \begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$, find $(A')^{-1}$

14. The monthly incomes of Mohan and Sohan are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves ₹ 15000/- per month, find their monthly incomes and expenditures using matrices.

15. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that (AB)′ = B′A′

16. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \ and \ x^2 = -1$

Then show that $(A + B)^2 = A^2 + B^2$

17. Prove that $aI + bA + cA^2 = A^3$, if $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$

18. If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, then find A^3.

19. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \ B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2 + 2AB$, find *a* and *b.*

20. If $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$, then find the value of a, b and c. Such that $A^T A = I$

21. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} a^n & b(\frac{a^n - 1}{a - 1}) \\ 0 & 1 \end{bmatrix}$, for all $n \in N$.

22. Find the value of k, if: $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

23. If x, y and z ∈R, and

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 2x & 5x+2y & 7x+5y+2z \\ 3x & 7x+3y & 9x+7y+3z \end{vmatrix} = -16, \text{ then find value of x.}$$

24. Find the value of 'k' if $\begin{vmatrix} 1 & a^2 & a^4 \\ 1 & b^2 & b^4 \\ 1 & c^2 & c^4 \end{vmatrix} = k \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Using properties of determinants, prove the following (Q. 68 to 31)

25. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = 0$

26. $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ac & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

27. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

28. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

29. $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

30. $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc-a^3-b^3-c^3)$

31. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

32. Given $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$. Find the product AB and also find $(AB)^{-1}$

33. Using properties of determinants, solve for x:

$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

34. If $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$ and $a \neq b \neq c$ then find the value of x.

35. Express the following matrix as the sum of symmetric and skew-symmetric matrices and verify your result.

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

36. If x = – 4 is a root of a $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0,$ then find the other two roots.

37. Using properties of determinants. Find the value of ‘x’

$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4-x & 4+x & 4+x \end{vmatrix} = 0.$

38. Using proportion determinants prove that

$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(2x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$

39. If $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$ using properties of determinant

Find the value of f(2x)-f(x)

40. If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ show that $A^2-5A+4I=0$.

Hence find A^{-1}

41. It $A=\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ show that $A^2=A^{-1}$

6 MARK QUESTIONS

1. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x + y + z)$ and hence find the quotient.

2. Using elementary transformations, find the inverse of the matrix

$$A=\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

3. Using matrix method, solve the system of linear equations

$x-2y=10, 2x-y-z=8$ and $-2y+z=7$

4. Find A^{-1} if $A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1}=\frac{A^2-3I}{2}$

5. Find the matrix x for which $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$, then show that $f(A) = 0$, using this result find A^5.

7. If $a + b + c = 0$ and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$, then show that either

$$x = 0 \qquad \text{or} \qquad x = \pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$$

8. If $A + B + C = \pi$, then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -sinB & 0 & tanA \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

9. If $\Delta = \begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ {}^{2} & (x+1)^2 & (x+2)^2 \end{vmatrix}$ prove that Δ is negative.

10. Using properties of determinants prove that:

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

11. Prove that: $\begin{vmatrix} a & a+c & a-b \\ b-c & b & b+a \\ c+b & c-a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

12 If a, b, c are p^{th},q^{th} and r^{th} terms respectively of a G.P. Prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

13. Prove that (x-2) (x-1) is factor of $\begin{vmatrix} 1 & 1 & x \\ \beta+1 & \beta+1 & \beta+x \\ 3 & x+1 & x+2 \end{vmatrix}$ and hence find the quotient.

14. Prove that:

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^{2)} & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

15. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

and use it to solve the system of equations:

$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1$

16. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the system of linear equations: $x + 2y + z = 4,\ -x + y + z = ,\ \ x - 3y + z = 2$

17. Solve given system of equations by matrix method:

$\frac{2}{a} + \frac{3}{b} + \frac{4}{c} = -3,\ \frac{5}{a} + \frac{4}{b} - \frac{6}{c} = 4,\ \frac{3}{a} - \frac{2}{b} - \frac{2}{c} = 6$

18. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap books and 28 pastel sheets. While school C sold 26 paper bags, 18 scrap books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

19. Two cricket teams honored their players for three values, excellent batting, to the point bowling and unparalleled fielding by giving ₹ x, ₹ y and ₹ z per player respectively. The first team paid respectively 2, 2 and 1 players for the above values with a total prize money of 11 lakhs, while the second team paid respectively 1,2 and 2 players for these values with a total prize money of ₹ 9 lakhs. If the total award money for one person each for these values amount to ₹ 6 lakhs, then express the above situation as a matrix equation and find award money per person for each value.

20. It A= $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ find A^{-1} using elementary transformation

Hence solve the system of linear equations.

x-2y=10

2x-y-z=8

-2y+z=7

ANSWERS

ONE MARK QUESTIONS

1. $\frac{1}{2}$

2. $\lambda = 4$

3. $A^2 = I_3$

4. $\alpha = a^2 + b^2, \ \beta = 2ab$

5. A

6. 0

7. 3

8. K=27

9. X = 5

10. a= $\frac{3}{2}, \ b = \frac{-2}{3}$

11. $|A| = 10$

12. $X = \begin{bmatrix} 0 & -4 & 1 \\ -3 & -3 & -7 \\ -5 & -1 & -5 \end{bmatrix}$

13. [20]

14. $\begin{bmatrix} 2 & -3 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -4 & -17 \\ 16 & 47 \end{bmatrix}$

15. $A^3 = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

16. 11

17. 0

18. $x = 0$

19. $|A| = 1$

20. $x = \frac{1}{2}(A + A'),\ y = \frac{1}{2}(A - A')$

21. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

22. $x = \pm 4$

23. 0

24. 9

25. –1

26. 405

27. 1

28. 0

29. 3 – 2x

30. 625

31. 729

32. 1728

33. 11

34. –11

35. $a^9,\ a^6$

2 MARK QUESTIONS

1 (a) $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ 1 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

4. x – y = –7

9. $\begin{vmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{vmatrix}$

10. 25

11. 0

12. $D_1+D_2=2$

14. $M_{11}=-12,\ M_{21},=-16,\ M_{31},=-4$
$C_{11}=-12,\ C_{21},=16, C31 = -4$

15. x=2,y=2

16. $C = \begin{vmatrix} 12 & 4/3 \\ 4 & -14/3 \\ 25/3 & 28/3 \end{vmatrix}$

17. No value of x, for which $A^2=B$.

18. $A= \begin{vmatrix} -1 & 5 \\ 1 & -2 \\ 3 & 0 \end{vmatrix}$

4 MARKS QUESTION

2. $x = 0$

4. $-5\sqrt{3}(5-\sqrt{6})$

5. $A^{-1} = \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$

6. $\begin{bmatrix} 5 & 0 \\ \frac{-6}{7} & \frac{4}{7} \end{bmatrix}$

8. $\frac{1}{10}\begin{bmatrix} 7 & -1 \\ -4 & 2 \end{bmatrix}$

9. x = 4

10. $B = \begin{bmatrix} 2 & -15 \\ 0 & -3 \end{bmatrix}$

11. $\propto = 5$

12. $X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

13. $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

14. Incomes: Rs 90,000/- and Rs 1,20,000/-

Expenditures: Rs 75,000/- and Rs 10,5000/-

18. $\begin{bmatrix} \cos 8\theta & \sin 8\theta \\ -\sin 8\theta & \cos 8\theta \end{bmatrix}$

19. a = – 1, b = – 2

20. $a = \pm\frac{1}{\sqrt{2}}$; $b = \pm\frac{1}{\sqrt{6}}$; $c = \pm\frac{1}{\sqrt{3}}$

22. K = 2

23. x =2

24. K =(a + b) (b + c) (c + a)

32. $AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$

33. x = 4

34. $x = \frac{-abc}{ab+bc+ca}$

35. $A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

36. x = 1,3

37. x=0,-12

39. ax (2a + 3x)

40. $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

6 MARK QUESTIONS

1. $(x + y + z)(xy + yz + zx - x^2 - y^2 - z^2)^2$

2. $A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$

3. x = 0; y= -5; z = –3

4. $A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

5 $x = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$

6. $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

8. 0

13. β

15. *Product* = 81

x = 3, *y* = -2, *z* = -1

16. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ $x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5}$

17. *a* = 1, *b* = – 1, *c* = – 2

18. School A = ₹ 850

School B = ₹ 805

School C = ₹ 970

19. Excellent batting: 3 lakhs

point bowling: 2 lakhs

fielding: 1lakh

20. $A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

x=0, y=-5, z=-3.